TMUA Paper 2 logic sheet

Keep this sheet next to you when you practise TMUA Paper 2 questions. It is based on the TMUA Notes on Logic and Proof.

Reading "if A, then B"

If A is true, B must be true. If A is false, the statement says nothing about B.

Phrase	Means
A is sufficient for B	If A, then B
A is necessary for B	If B, then A
A only if B	If A, then B
A if B	If B, then A
A if and only if B	Both directions are true

Converse, contrapositive and inverse

Start from "if A, then B".

Name	Statement	Same truth as original?
Converse	If B, then A	Not always
Contrapositive	If not B, then not A	Always
Inverse	If not A, then not B	Not always

Example: "if n is a multiple of 6, then n is even" is true. Its contrapositive, "if n is odd, then n is not a multiple of 6", is also true. Its converse, "if n is even, then n is a multiple of 6", is false (n = 2).

Counterexamples

- One counterexample is enough to show a "for all" statement is false.
- No number of examples proves a "for all" statement.
- One example is enough to prove a "there exists" statement.

Common errors in proofs

- Dividing both sides by something that could be zero.
- Squaring both sides, then treating every answer as a solution of the original equation.
- Assuming the thing you are trying to prove.
- Treating a necessary condition as if it were sufficient.
- Checking a few cases and calling it a proof.

Checklist for a Paper 2 argument question

1. Read the claim and decide what would count as proving it.
2. Check each line follows from the lines before it.
3. Ask whether any step only works in one direction.
4. Test the final answer in the original statement.
5. Read every option to the end before choosing.